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# The Study of the Nucleus as a Theme in Contemporary Physics

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## Introduction

Niels Bohr was just eleven years old when Becquerel discovered the first hint of the existence of the atomic nucleus in those faintly glowing ashes from the ancient cosmic fireworks that created the elements of which our solid earth and our living bodies are made; fifteen years later Rutherford was to exploit the natural radioactivity as a marvelous extension of the human sense organs able to resolve the atomic systems into their open, planetary electronic structures surrounding the dense, small, enigmatic atomic nucleus. This report will be an attempt to characterize and review the shifting questions and the physical problems that motivated them in the studies that have led to a growing understanding of this unexpected new form of matter occurring in atomic nuclei. I hope also to be able to indicate how some of these histo-ically important issues have reappeared, transformed, as central issues in esearch. In attempting to retell some of this history I cannot avoid a curren concerr, with the presumptuousness of doing this in front of an audience that includes some of the most important leaders and contributors to these developments. It may be a partial extenuation for me to admit that I see this as a chance to report how the historical tradition has been transmitted and understood by this particular member of the younger generation, and to strongly encourage you of the heroic generation to correct me and to bring your own witness where my interpretation seems to you to be inappropriate.

To indicate the broad structure of the development I find it useful to recognize in the history of nuclear physics three periods distinguished by the rather different character of the central question being asked.

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(1) The discovery of the nucleus and its constituents (1886–1935). The achievement of this period was the identification of the "nuclear problem" as involving a composite non-relativistic quantal system built out of neutrons and protons, held together by a new force of nature—the "strong" interaction.

(2) *Defining the nuclear paradigm* (1935–1952). The developments of this period led to the recognition of nuclear structure as based on independent-particle motion capable of supporting a rich variety of collective dynamics.

(3) Discovering the feel of the nuclear stuff (1948-present).

The main focus of the present report is period 2, but I shall attempt to put the issues in a broader perspective by including some of the background from period 1 and the further development occurring in period 3.

### 1. The discovery of the nucleus and its constituents

Rutherfords discovery revealed the nuclei as a new constituent of matter.

To begin with, the different nuclei appeared as "elementary particles", in the sense that there did not appear to be any more *a priori* reasons for the existence of any one of these nuclei than there was for the existence of the electron. However, as is the way with "elementary particles" it gradually became apparent that the nuclei formed a large, but strongly ordered, family, and there accumulated compelling evidence for the view that the nuclei are composite systems built out of more elementary constituents.

(i) Radioactivity itself revealed the possibility of transition from one member state to another of the nuclear family. In particular,  $\alpha$ -decay suggested the possibility of  $\alpha$ -particles as potential constituents of nuclei.

(ii) The quantization of nuclear charge (Moseley 1913) and approximate quantization of nuclear mass [Prout's hypothesis (1815), enormously strengthened by the work of Aston (1920)] suggested that nuclei are composed of a discrete number of fundamental building blocks.

(iii) The discovery of induced nuclear reactions (Rutherford 1919) and artificial radioactivity (Joliot and Curie 1934) directly exhibited the possibility of changing and exchanging the elementary building blocks in nuclear processes.

Despite these significant clues, the construction of nuclei out of the then known particles, electron and proton, posed profound problems and, indeed, seemed to link the nuclear problem with the unsolved problems of relativistic quantum theory. The sense of confusion and mystery at this time is strikingly expressed in Niels Bohr's Faraday lecture (held in 1930 and published in 1932):

"Still, just as the account of those aspects of atomic constitution essential for the explanation of the ordinary physical and chemical properties of matter implies a renunciation of the classical ideal of causality, the features of atomic stability, still deeper-lying, responsible for the existence and the properties of atomic nuclei, may force us to renounce the very idea of energy balance. I shall not enter further into such speculations and their possible bearing on the much debated question of the source of stellar energy. I have touched upon them here mainly to emphasize that in atomic theory, notwithstanding all the recent progress, we must still be prepared for new surprises."

We can now see that this situation was almost inevitable until the neutron had been discovered. Telescoping a marvelous scientific adventure into a mere telegraphic report we may remember that this discovery came from:

(i) Rutherford (1920) predicted the existence of the neutron (in his Bakerian lecture) by arguing that if heavy nuclei could form tightly bound states with electrons as revealed in the difference between the atomic number A and the positive charge number Z, one could very well expect a single proton to unite with a single electron to produce a neutral and very unusual nuclear system. He felt, also, the need for such neutral nuclear systems in order to account for the building up of the heavy elements. This vision by Rutherford appears to be the first successful prediction of an elementary particle.

(ii) *Chadwick* joined Rutherford (1920–1932) in a wide-ranging research program aimed at producing and exhibiting the expected neutron.

(iii) Bothe and Becker (1930) observed a penetrating radiation produced in Be + He reactions and interpreted this as a high-energy  $\gamma$ -ray. Joliot and Curie (1932) observed that the new radiation produces energetic recoils when passed through paraffin, but continue to interpret the radiation as a high-energy  $\gamma$ -ray.

(iv) Chadwick (1932) compared the recoils in H, He, and in N, to determine the mass of the new radiation and found  $M_{\rm rad} \approx M_{\rm prot}$ , and thus the neutron is discovered at last!

After the discovery of the neutron Heisenberg (1932), Majorana (1933) and Wigner (1933) took the first steps to pursue the consequences of this discovery with respect to the nuclear problem. Their program can be briefly summarized:

(i) Nuclei are composite systems built out of neutrons and protons. This picture provided an immediate interpretation of the integer quantization of nuclear charge and mass:

Z = number of protons,

A = number of protons + number of neutrons.

(ii) The nuclear binding required a new force of nature (which we now recognize as the first example of the "strong" interaction). A number of significant features of this interaction could be derived from the available systematics of nuclear binding energies:

-saturation (binding proportional to A);

-strong force (nuclear binding is of order 10<sup>6</sup> stronger than atomic binding);

-charge symmetry (from  $A \approx 2Z$ , with A - 2Z increasing with Z as a result of Coulomb repulsion);

*-finite range*  $\sim 2 \times 10^{-13}$  cm (from comparison of <sup>2</sup>H and <sup>4</sup>He binding).

The final resolution of the questions of period 1 had still to wait two years until Fermi (1934a) developed the theory of  $\beta$ -decay exploiting the freedom provided by the quantal formalism to have the electron and neutron created at the instant of the decay process. At last the nuclear dynamics could be totally freed from the terrible consequences of trying to think of bound electrons inside the nucleus.

Before leaving the achievements of period 1, and looking at the description of nuclei considered as built out of neutrons and protons, I would like to remind you that already at this juncture there began to appear, at first obscurely, but with constantly growing insistence, significant results that indicated the limitations of this picture and pointed toward the composite nature of the neutrons and protons themselves. Of course, the very existence of two states of the nucleon, the neutron and proton, can be seen (at least today) as a strong hint of internal structure, and then the discovery of the anomalous magnetic moment of the proton (Estermann and Stern 1933, Frisch and Stern 1933) should have removed all doubt about the elementarity of these particles. Finally Yukawa's invocation of massive quanta as the mediators of the strong interaction (Yukawa 1935) provided an energy scale setting the limits beyond which the compositeness of the nucleons would have to be seriously taken into account.

The question of the proper place of these "additional" degrees of freedom in the problems of nuclear structure has been a recurring theme and is currently a focus of active interest. Let me remind you of a rather extreme view considered by Niels Bohr (apparently sometime in the late 1930s). According to J.H.D. Jensen (1965), Bohr argued that

"... since the field is strongly coupled to its sources, the hitherto existing picture of the 'compound nucleus' may still be much too naive. Perhaps, the only sensible concept is to consider the whole nucleus as an 'Urfeld' which is highly non-linear because of such strong couplings. When this field is quantized it must give (in addition to other conserved quantities, like angular momentum) integral charges Z, and energies (i.e. masses) that form a spectrum with values close to the integral numbers A, on which the 'excitation energy' bands are superposed. The assumption that inside the nucleus there exist Z protons and A - Z neutrons, such as we encounter them as free particles in appropriate experiments, would then hardly make any sense." \*

As I mentioned, these questions, slightly reformulated, are under active current investigation. Let me attempt a capsule assessment of the present status of these issues:

(i) The exchange of mesons between nucleons implies modifications in the electromagnetic and weak decay-properties of nucleons in the nucleus as compared with that of a collection of free nucleons. These modifications are relatively small in low-energy transitions, typically of order 10%, but in favourable cases they have been quantitatively identified. \*\* Note that this figure of 10% also represents the accuracy of the non-relativistic approximation in nuclei,  $(v/c)^2$ , as well as the ratio of the  $\pi$ -meson to nucleon mass, but I do not know a satisfactory general argument establishing a connection between these numbers.

(ii) Recent experimental studies of collective spin-excitations in nuclei (the so-called Gamow-Teller resonances) have revealed rather narrow and well-defined collective vibrational modes excited in high-energy proton-neutron (pn) reactions. [Goodman (1984) and fig. 1.] The absolute cross-sections for excitating these

<sup>\*</sup> It is likely that it is this picture that is being referred to in letters by Rutherford in which he talks of Bohr's view of the nucleus as a "mush of particles of unknown kind, the vibrations of which can be deduced on quantum ideas" [Rutherford's letter to Born (1936), published in "Niels Bohr, Collected Works", Vol. 9 (Peierls, 1985)]. Indeed Bohr himself, in his compound-nucleus article in Nature (1936) refers obliquely to the possibility of this picture.

<sup>\*\*</sup> An especially well-studied case is provided by the neutron-proton capture reaction [Riska and Brown (1972); for a review of the status of exchange effects in heavier nuclei see Yamazaki (1979) and Arima and Hyuga (1979)].



Fig. 1. Excitation of nuclear collective spin-isospin resonance. The figure exhibits the yield of the proton-neutron (p n) reaction at 0° for 200 MeV protons incident on <sup>208</sup> Pb (Goodman 1984). The strong peak with  $Q \approx -20$  MeV, corresponds to a collective excitation produced by the "Gamow-Teller" operator  $\tau_+ \sigma$ .

resonances are about a factor of two less than predicted, assuming a simple mean-field description based on neutrons and protons. A significant part of this missing strength can be attributed to the effect of the spin-dependent nuclear mean-field acting on the spins of the quarks within the nucleons causing excitation of the  $\Delta$ -resonance. It has not yet been possible to quantitatively determine the magnitude of this effect because of uncertainty concerning the line shape of the resonance. However, the nuclear-physics tools at present available should make it possible to settle this question and thus establish a quantitative measure of the role of the  $\Delta$ -degree of freedom in this particular nuclear process.

(iii) A much more profound effect of additional degrees of freedom in nuclear matter would follow from conjectures perhaps suggested by quantum chromodynamics and bag models. The interpretation of quark confinement as an effect of the QCD vacuum acting as a medium from which color is excluded, has led to the suggestion that at sufficiently high energy-density (high temperature and/or baryon density) nuclear matter will exhibit a phase with unconfined color [see the review by Jacob and Van (1982)]. Here we would indeed encounter a phase of matter resembling that in Bohr's vision quoted above. The attempts to make quantative estimates of the energy density necessary in order to produce this new form of matter are still rather uncertain but indicate something like a doubling of the energy density as compared with the equilibrium state of nuclear matter. A possible environment for realizing such energy densities may be provided by collisions between heavy nuclei involving bombarding energies in the range 10 to 100 GeV/nucleon. There are intensive efforts to explore possibilities for creating matter under these conditions and to attempt to find diagnostic signals which would make it possible to probe the equation of state describing this regime.

#### 2. Defining the nuclear paradigm

After the discovery of the neutron and Fermi's formulation of  $\beta$ -decay, it became possible to begin considering the dynamical patterns and structures formed by the neutrons and protons of the nuclei. The subsequent developments were strongly driven by the experimental discoveries that were constantly revealing new features of the nuclear systems. The beginning of the period saw the first nuclear reactions produced by artificially accelerated particles, as well as the use of the recently discovered neutron as a projectile capable of penetrating to even the heaviest nuclei and causing reactions. These neutron reactions, especially developed and exploited by Fermi and his collaborators, were uniquely important in focussing attention on the many-body aspects of the nuclear problem. Let me again resort to a telegraphic style to remind you of the bare outlines of the development:

(i) Early 1934, Fermi and his collaborators begin systematic neutron irradiation of all elements of the Periodic System, and find new radioactivities in most of them (Fermi 1934b, Fermi et al. 1934a).

(ii) October 1934, discovery of added effectiveness of slow neutrons (Fermi et al. 1934b).

(iii) Through the year 1935, theoretical analysis of neutron reactions on basis of particle motion in a static-potential model (Fermi and Rasetti 1935, Bethe 1935, Perrin and Elsasser 1935, Beck and Horsley 1935). Main results:

- $-\sigma_{n\gamma} \sim \frac{1}{v}$  at low neutron-energy,
- Short residence time of neutron in nucleus implies monotonic cross-sections in energy region below  $\sim 1$  MeV,
- $-\sigma_{cap} < \sigma_{scat}$  in all cases.

It is of some importance for assessing the frame of mind at this time to attempt to understand the motivation and degree of conviction with which the static-potential model was being used. It appears that the model was mainly motivated by its successes in describing collisions of electrons with atoms and, recognizing the great differences between atoms and nuclei, the model was being used without any great conviction; for example, near the end of his article Bethe writes:

"It is not likely that the approximation made in this paper, i.e. taking the nucleus as a rigid body and representing it by a potential field acting on the neutron is really adequate ... Anyway it is the only practicable approximation in many cases ..."

(iv) Also in 1935, large capture cross-sections observed for some elements\* and discovery of sharp resonances (Tillmann and Moon 1935, Bjerge and Westcott 1935) called "selective absorbtion", being in violent disagreement with the theory in (iii) and provoke the formulation of "compound nucleus" (Bohr 1936).

There exist published reports by J. Wheeler (1979) and by O.R. Frisch (1967) colorfully recounting discussions at the Niels Bohr Institute during the time the

\* I am endebted to Professor Amaldi for pointing out to me that it was Dunning et al. (1935) who first established  $\sigma_{abs} \gg \sigma_{scat}$  for slow neutrons on Cd.

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Fig. 2. Bohr's picture, visualizing the formation of a compound nucleus by the capture of a neutron.

compound nucleus was being formulated. I shall not repeat these accounts here but would like to go directly to an examination of the content of Bohr's analysis.

The core of Bohr's thinking is the recognition that the densely packed nuclear system being studied in the neutron reactions, forces one to place the collective many-body features of the nuclear dynamics at the center of attention. To illustrate these ideas I do not know of any better figures than those prepared by Niels Bohr in connection with lectures which he gave at that time and which were published in the same issue of "Nature" (as a new item) that contains his famous article \*. The first (fig. 2) draws attention to the far-reaching consequences for the course of a nuclear reaction, of the assumption of a short mean free path for nucleons in the nucleus.

If we imagine the balls removed from the central region of the figure, the ball entering from the right will be accelerated as it enters the central depression, but just this acceleration ensures that, after running across to the opposite side, the ball will have enough energy to surmount the barrier on that side and run out of the nuclear region.

A very different dynamical evolution results if we restore the balls to the central region. Now, the entering nucleon will soon collide with one of the nucleons of the target and, sharing its energy with the struck nucleon, will no longer be able to leave the confining potential. Being reflected back it will collide and share its remaining energy with still other nucleons; these struck nucleons will also collide and ultimately the total energy will be distributed among all the nucleons in a distribution of the type described by the equilibrium distribution of the kinetic theory of gases. In this situation the only possibility for one of the nucleons to escape from the central region requires the occurrence of a fluctuation in which almost all of the energy is again concentrated on a single particle, which will then be able to surmount the confining potential. The unlikelyhood of such an extreme fluctuation implies that the duration of the reaction phase is enormously increased (as compared with the first situation considered with only a static potential acting).

This increase of reaction time makes it possible to explain both the observed large ratio of capture to scattering cross-sections for slow neutrons as well as the

<sup>\*</sup> Apparently the original draft of these figures was executed by O.R. Frisch for Niels Bohr (Frisch 1979).



Fig. 3. Bohr's sketch of a schematic nuclear level spectrum. The dashed line indicates the neutron-binding energy.

narrowness of the selective absorption bands. Perhaps even more important, the intermediate stage representing a kind of thermal equilibrium from which the final decay represents a rare fluctuation, ensures that the relative probability of different final states will be governed by statistical laws and is independent of the mode of formation of the compound system.

Figure 3 shows Bohr's sketch of a schematic nuclear level spectrum. The study of radioactivity had shown that the lowest states in heavy nuclei have excitation energies on the order of a fraction of 1 MeV, and Bohr assumed that these excitations represent some sort of collective vibration of the whole nucleus. With increasing excitation energy an increasing number of different vibrational modes can be excited and the different possibilities for partitioning the total excitation energy between these different modes leads to an enormous increase in the total number of excited states. All of these quantum states can be resonantly excited by an incoming neutron, thus accounting for the dense spacing and narrowness of the levels observed in the selective absorption phenomena. The dotted line in the magnifying glass at about 10 MeV indicates the neutron separation energy, but the

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level scheme above and below this line are not significantly different; indeed, the neutron-escape probability is much less than the  $\gamma$ -emission probability for levels slightly above this energy as a result of the extreme improbability of the fluctuation required to concentrate all of the excitation energy on a single particle. Only at higher energies will the neutron-emission probability contribute appreciably to the width of the individual levels and lead eventually to a smearing out of the spectrum (indicated in the upper magnifying glass at about 15 MeV). Bohr contrasts this picture of densely-spaced many-particle levels in the nucleus with the spectrum of atoms excited in collisions with electrons, where the incident electron will at most collide with one of the atomic electrons causing it to change its binding state from one orbit to another.

The profound reordering of the picture of nuclear dynamics implied by Bohr's ideas was, apparently, rapidly and widely accepted in the nuclear physics community; within months the literature is dominated by papers applying, testing and extending the ideas of the compound nucleus.

In view of the subsequent history, it is an interesting and relevant question to ask whether Bohr's vigorous and effective contribution to the development of nuclear physics at this time had also an adverse element in preventing an earlier appreciation of the significance of independent-particle motion in the nucleus. The tentative use of an independent-particle picture has been mentioned above but after Bohr's paper such approaches were subjected to a much more critical attitude. The independent-particle starting point was further developed, especially by Feenberg and Wigner (1937) and by Rose and Bethe (1937) as a basis for the analysis of the configurations of light nuclei \*, but as Maria Goepert Mayer (1964) says in her Nobel Lecture:

"[the model] failed in predicting the properties of heavy nuclei, and somehow, the theory of individual orbits in the nucleus went out of fashion."

We may ask, did this going out of fashion delay the understanding of nuclear properties? To what extent was it a psychological question connected with Bohr's enormous prestige? As Victor Weisskopf remarked to me once when discussing this question, "You know, it wasn't easy to disagree with Niels Bohr".

It is in the nature of these questions that the answers can only be tentative and partial, but my impression is that the direction of the development of nuclear physics at this time was strongly bound to the available experimental tools and the limited number of facts about the nucleus that were then accessible. The discoveries that were being made focused attention mainly on a variety of reaction processes for which the compound nucleus was the uniquely appropriate and powerful concept. One can ask, what properties would one have understood better by invoking individual orbits? What data could have been used to test that idea? Only at a much later stage with the accumulation of more detailed and systematic knowledge on

<sup>\*</sup> In the literature of that time it is stated again and again [see Wigner (1933), Bethe (1935, 1936)] that the independent-particle picture will not be applicable to heavy nuclei but might be appropriate for lighter systems. I am unable to discover, or understand, the basis for this expectation of a difference in the dynamics of light and heavy nuclei.

nuclear masses, spins, moments and excitation spectra could there be a proper assessment of the role of single-particle motion. Having said this, however, I think it is also relevant to notice that when that time came, the decisive contributions were made by scientists who were in a significant sense outsiders to the main development of the field; Hans Jensen in the scientifically isolated conditions of postwar Germany, and Maria Mayer, a chemical physicist newcomer to the field of nuclear physics, could look at this new data with uniquely creative vision. It might appear that by this time, ten years after the formulation of the idea of the compound nucleus, the successes of this idea had induced a certain orthodoxy such that most of the established figures were inhibited in reading the message contained in the burgeoning new facts about the nuclei.

I would also like to express the opinion that the close connection between experimental initiative and theory building, to which I referred above, has continued to be characteristic of the most fruitful developments in nuclear structure—one is almost tempted to say of fruitful developments in all those parts of physics dealing with systems with many degrees of freedom.

But now I would like to return to the early period after the formulation of the compound-nucleus idea and, again in a telegraphic style, remind you of the impressive series of developments in which this idea was extended, and successfully applied to the interpretation of the growing body of knowledge about nuclei (see table 1). For the first ten years after its formulation the compound nucleus served brilliantly as a basis for relating and interpreting the experiments that were gradually probing more and more deeply into the facets of nuclear structure. I do not know of any significant criticisms during that time of the assumptions of the compound nucleus or challenges to its explanatory power. Especially in the study and interpretation of the many phenomena associated with the fission reaction, the compound nucleus, coupled with the analogy of nuclear matter to that of a liquid drop, provided a marvelously successful conceptual basis.

Then, as is well known, there came a second major reordering of the picture of nuclear structure as it was recognized that a wide variety of nuclear systematics (mainly referring to binding energies, but also extending to the data on nuclear spins, magnetic moments, the occurrence of isomerism, etc.) testified to the existence of nuclear shell structure, i.e. independent-particle orbits as a basis for the nuclear ground states (Mayer 1948). This discovery carried a strong sense of paradox that is preserved in the early reference to the closed shells as "magic numbers" (an expression coined by Wigner). The paradox, of course, resulted from the fact that independent-particle motion seemed to be incompatible with the ideas of the compound nucleus.

At first, it was suggested that the shell structure might, in some way, be confined to the ground state while the compound-nucleus ideas would describe the excited states of the nucleus. But then Barschall (1952) pointed out that the neutron cross-sections (averaged over individual resonances) for incident neutrons of energy 0-3 MeV showed systematic variations (see fig. 4) that were in striking disagreement with the universal and monotonic pattern expected if the mean free path for the neutron in the nucleus would have been very short ("black nucleus"). This data were then interpreted by Feshbach, Porter and Weisskopf (1953) in terms of an

Development	Parameters	References
Resonance formula	$\sigma_{n\gamma}(E) = \pi \lambda^2 \frac{\Gamma_{\gamma} \Gamma_n}{\left(E - E_0\right)^2 + \left(\frac{1}{2} \Gamma_{\text{tot}}\right)^2}$	Breit and Wigner (1936)
Level density and thermody-	entropy: $\alpha \ln \rho$	Bethe (1936)
namic concepts	temperature: $\frac{1}{T} = \frac{1}{\rho} \frac{\mathrm{d}\rho}{\mathrm{d}E}$	Bohr and Kalckar (1937)
Nuclear decay as evaporation	reciprocity arguments	Weisskopf (1937)
Cross-sections for "black" nucleus		Bethe (1940) Feshbach, Peaslee and Weisskopf (1947)
Semi-empirical mass formula	bulk energies (volume, surface, symmetry) pairing energy	Weizsäcker (1935)
Collective vibration of nucleus	shape oscillations density fluctuations	Bohr and Kalckar (1937)
		Migdal (1944)
	electric dipole mode	Baldwin and Klaiber (1947) Goldhaber and Teller (1948)
Fission: The compound nucleus		Hahn and Strassmann (1939)
finest hour!		Meitner and Frisch (1939)
		Bohr and Wheeler (1939)

Table 1 Major developments bearing on compound nucleus (1936-1948).



Fig. 4. Systematics of neutron total cross-sections for  $0 \le E_n \le 3$  MeV (Barschall 1952).

Table 2				
Time scales	in	nuclear	reactions.	

Aspect	Time scale	
Traversal time, $\tau_0$	$\tau_0 = \frac{2R_0}{v_{\rm in}} \sim 10^{-22} {\rm s}$	
Collision time, $t_{col}$ ( = mean free path/ $v_{in}$ )	$t_{col} = \frac{\hbar}{w} \sim 6 \times 10^{-22} \text{ s}$ w = absorption potential ~1 MeV	
Single-particle residence time, $t_{in}$	$t_{in} = \tau_0 / T$ T = transmission coefficient of nuclear surface $\sim (\frac{\lambda_{\text{out}}}{\lambda_{in}})$ $\sim 10^{-4}$ slow neutron $t_{in} \sim 10^{-18}$ s	
Physical pictures	"black nucleus": $t_{col} < \tau_0$ shell structure: $t_{col} > \tau_0$ compound nucleus: $t_{col} < \tau_{in}$	

"optical" potential in which the neutron mean free path for absorption was  $\sim 20$  fm. Thus, the assumption that the mean free path was short compared to nuclear dimensions, believed to be a cornerstone of the compound nucleus, was shown to be wrong! But still the compound nucleus has survived and continues to be the basis for interpreting a large part of the data on nuclear reactions. The resolution of this paradox is provided by a more careful examination of the characteristic times involved in different nuclear processes. (See table 2.)

The necessary conditions for the occurrence of shell structure (and for systematics of the type pointed out by Barschall) is  $t_{col} > \tau_0$ , but the condition for formation of the compound nucleus is  $t_{col} < t_{in}$  and thus both of these conditions are well satisfied. It is the strong reflection of slow neutrons at the nuclear surface that extends the residence time so effectively and makes the subsequent history of the reaction very sensitive to the rather weak coupling of the projectile to the complicated motion of the compound nucleus. The weakness of this coupling is revealed only in the somewhat detailed features of nuclear reactions, such as the relative narrowness of strength functions and other phenomena that measure the residual features of single-particle motion surviving in the compound nucleus somewhat like the smile that still remains after the disappearance of the Cheshire cat.

I would like to emphasize that the residence time of slow neutrons in the nucleus exceeds the traversal time by such a large factor that the compound nucleus would continue to be the crucial concept in the analysis of neutron reactions, even if the mean free path for energy exchange would have been appreciably longer than the observed value; the co-existence of independent-particle motion and the many-body phenomena of the compound nucleus is thus *not* an uncanny accident hinging on a fine balance in the parameters of the nuclear interactions, but appears to be a rather general feature that is expected in wide classes of quantal systems.

If we now look back over the development of nuclear physics in the period 1933-1952, we see, besides the great discoveries of different types of nuclear reactions and processes, a gradual clarification of the nature of that fascinating new form of matter encountered in nuclei. A deep understanding of the dynamics of this matter could not be built until one had settled on the correct starting point: Is one to start from something like the localized highly correlated picture of a solid, or from the delocalized orbits of particles quantized in the total volume of the nucleus? The question is, of course, intimately linked to the strength of the nuclear forces (measured in units of the Fermi energy which is a measure of the energy required to localize particles at the equilibrium density). From this point of view one may feel that from the start there were strong arguments to believe that the forces are rather weak—in the two-body system there is only one very weakly bound state for T=0and no bound state at all for T=1—and thus unable to produce the localization necessary for a quantum solid. We must, however, remember that in assessing this question today we are exploiting the results of a long development in which the analysis of nuclear matter could be compared with a variety of quantal systems encountered in condensed-matter physics and that even with this advantage the answers are not very simple (see, for example, the necessary uncertainty in discussing the deconfinement transitions for quarks and gluons, as well as the question of a possible solid phase in the interior of neutron stars). We are here forcefully reminded that despite the impressive development of the powers of formal analysis, the important many-body problems of nature have repeatedly revealed the deepseated limitations of straightforward reductionism. Each rung of the quantum ladder has revealed marvelous structures, the interpretation of which has required the invention of appropriate concepts which are almost never discovered as a result of purely formal analysis of the interactions between the constituents.

## 3. Discovering the feel of the nuclear stuff

The recognition of single-particle motion in the average nuclear potential provided a basis for developing a very detailed understanding of the nuclear dynamics, an understanding that reveals a fascinating tension between the concepts relating to independent-particle motion and those referring to collective features associated with the organized dynamics of many nucleons. I shall not attempt here to even enumerate in any systematic manner the rich variety of phenomena that have been revealed by these studies. Rather, I shall complete the present report with a few remarks on the further evolution of the compound-nucleus idea in connection with the statistical theory of quantal spectra, a development that will have to serve as a single illustrative example exhibiting some of the features of style and perspective characteristic of the third historical period of nuclear studies.

The experimental impetus for this development is again the neutron resonances which played such an important role in the original inspiration of the compoundnucleus idea. It is impossible for me to think about these resonances without a sense of awe at the profound generosity of nature in providing a window in the nuclear spectra at a point where the level densities are about a million times greater than

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those of the fundamental modes; where the quantal levels are still beautifully sharp in relation to their separation, and where the slow neutrons provide an exquisitly matched tool with which to resolve and measure the properties of each resonance. The effective exploitation of this tool has provided complete spectra comprising hundreds of individually resolved and measured neutron resonances, while corresponding developments in charged-particle spectroscopy have led to the measurement of similar spectra for proton resonances. It was Wigner (1955) who initiated thinking about this material in terms of random matrices.

The idea is to use statistical arguments in order to characterize the wavefunctions and spectra describing the quantal spectrum of the compound nucleus. The compound-nucleus idea implies that the quantal states are complicated mixtures involving all the available degrees of freedom of the many-body system (something like ergodic motion in classical mechanics). Wigner suggested that significant features of these spectra might be modeled by considering, for some region of the spectrum, an expansion of the Hamiltonian matrix on an arbitrarily chosen finite set of basic states. The strong mixing of different degrees of freedom and the randomness of the compound nucleus is expressed by chosing the elements of the Hamiltonian matrix independently and randomly from an appropriate ensemble. We may then ask

- 1. Object of study (Wigner 1955):
  - (i) an ensemble of real orthogonal  $N \times N$  matrices (symmetry of H)
  - (ii) invariance of ensemble under orthogonal transformation (independence of choice of basis)
  - (iii) matrix elements independent random variables, (expressing "randomness" and strong coupling)  $P(H) = \operatorname{norm} \{ \exp(-C \operatorname{Tr} H^2) \}$   $C = \operatorname{constant}$  related to level density.
- 2. Transform to variables:

 $E_i$ , i = 1, ..., N, the eigenvalues  $X_i = \frac{1}{2}N(N-1)$ , "other" variables describing the eigenfunctions  $P(E_1...E_N) = \text{norm. } \Pi | E_i - E_j | \exp(-C \Sigma E_i^2).$ 

- 3. Note (Dyson 1962) that the probability distribution of the eigenvalues is identical with the partition function for N particles moving in 1 - d and interacting with
  - (i) an average confining potential  $U = -\sum x_i^2$ ,
  - (ii) a repulsive two-body force

 $V_{12} = -\ln|x_1 - x_2|.$ 

4. The analogy in 3 provides a physical picture for unstanding the "repulsion" of levels: (i) nearest-neighbor spacings S, approximately described by the "Wigner distribution":

$$P(S) = \frac{\pi}{2D^2} S \cdot \exp\left(-\frac{\pi}{4} \left(\frac{S}{D}\right)^2\right).$$

(ii) suppression of long-range fluctuations (screening)  $N(L) \equiv$  number of levels in interval L

 $\sigma_L^2 \equiv \text{mean square fluctuation in } N(L)$  $= \frac{2}{\pi^2} \ln(\frac{L}{D}) + \text{const.}$ 



Fig. 5. Level statistic  $\overline{\Delta}_3(\overline{n})$ . The quantity plotted is the mean square fluctuation in the number of nuclear levels included in an energy interval of a length such that  $\overline{n}$  levels would be expected in the average. The experimental data on neutron resonances is compared with the prediction based on the eigenvalues of random orthogonal matrices (Haq et al. 1982).

whether there are significant features in the eigenvectors and the eigenvalues which reflect the strong coupling of the different parts, but are otherwise universal in the sense of being the same for almost all of the matrices generated by this process.

It turns out that the answer to this question is: yes; indeed, as shown by Thomas and Porter, Mehta, Dyson, and French and coworkers, the *fluctuations* in level widths and spacings are just these universal properties [see table 3 and the review article by Brody et al. (1981)]. The extensive evidence from nuclear resonances referred to above has in recent years been shown to agree in striking detail with the prediction concerning these fluctuations based on random matrices (see fig. 5) and thus to confirm the applicability of this characterization of quantal states of the compound nucleus in the regions to which it has been applied. [These ideas have also been invoked in the interpretation of experiments on laser excitation of polyatomic molecules (Abramson et al. 1985, Sundberg et al. 1985) and in the discussion of electronic properties of small metallic particles (Gorkov and Eliashberg 1965).]

While the original formulation of this model was based on random matrices, current developments have made it possible to relate these characteristic features of quantal chaotic motion to more physical models [first to a model of electron motion in a disordered medium (Efetov 1983) and quite recently to direct semi-classical quantization of the classical chaotic motion based on the unstable periodic orbits (Berry 1985)].

#### B.R. Mottelson

The current questions are concerned with issues such as: How can one characterize the transition between the low-energy spectrum with its many conserved quantum numbers (classically multiply-periodic motion) and the compound-nucleus region, exhibiting quantal chaos? And how can one characterize the limitations on the random-matrix model that are associated with the existence of a finite relaxation-time for the nuclear configurations? These issues of chaos in quantal systems are a fascinating chapter in the continuing efforts to digest the significance of the quantal concept. For the nuclear physicist the compound nucleus provides a powerful inspiration in the struggle to understand this issue.

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#### Discussion, session chairman S. Belyaev

Weisskopf: I was very much impressed by your presentation of a period which I experienced here in this place in the most delightful and most exciting way. I know that you and I have discussed the question before whether Niels Bohr has retarded the development of the shell model or not. Of course, one should never say something negative about a person at his 100th birthday, and I am far from saying anything negative about a man who formed my life and my thinking. However, in some ways you have actually supported my remark, which I have made several times, that the tremendous personality of Bohr has steered our thinking in certain directions. You made yourself the remark that the shell model was actually introduced by outsiders. Now, perhaps this is not quite true. The shell model was brought to Chicago by Enrico Fermi, whom you can hardly consider as an outsider in any part of physics, and he actually induced and encouraged Maria Goeppert-Mayer to investigate these phenomena. It is true that at that time there was a lot of experimental material available to support the shell model, but I think that magic-number effects were already known before, but were not exploited. This may

have been caused by the tremendous—I would not say influence of Niels Bohr—but by the tremendous success of the compound-nucleus picture, which opened up so many new perspectives, including fission, that you call the finest hour of that picture. I would call it the most tragic hour.

*Peierls*: We should not exaggerate the responsibility of Bohr, through his authority, in delaying the study of the shell model. I must admit that I belonged to those who were convinced that the shell model could not work. This view started with the success of Bohrs compound-nucleus picture, but we then looked very seriously and quantitatively at properties on which the validity of the shell model would depend, and convinced ourselves about its impossibility. Our arguments were misleading for a number of subtle reasons, but they did not rely on Bohrs authority. I simply want to say that many of us shared the responsibility for maintaining Bohrs original view longer than it should have been.

Kohn: You indicated that the criteria for nuclear single-particle motion (the shell model) and of the compound-nucleus picture could be explained by ratios between characteristic times, and that these ratios are both of the order of  $10^4$ . What happens with these estimates for really small nuclei where, as far as I know, the compound-nucleus model is not very useful?

*Mottelson*: There still is enormous difference between the wavelength of the neutron outside the system, where its energy may be on the order of an eV, and the wavelength it has inside the nucleus. So there are still strong reflection effects for slow particles entering also light systems. That corresponding factor is also part of the description of nucleon capture by light nuclei.

Amaldi: I am really very much impressed by the capacity of Mottelson to summarize in three quarters of an hour the essential developments of such a long period. I would, however, like to make two minor remarks of historical nature. You have correctly said that one of the facts that led to the development of the compound-system model by Bohr was the fact that the large capture cross-section was not accompanied by a large scattering cross-section. It should be mentioned that this was proved experimentally by a group at Columbia University. The people were J.R. Dunning, G.B. Pegram, G.B. Fink and D.P. Mitchell, who published a paper in Physical Review in the summer of 1935. You also correctly mentioned the paper by Bjerge and Westcott (1935), and by Tillman and Moon (1935). They were the first to observe that the neutron-capture cross-section does not show the same dependence upon velocity in the different elements as would be necessary if the 1/v law had general validity. The fact that the cross-section was changing rapidly was then shown by Fermi and me some months later. In the winter of 1935-1936 we measured the width of the resonances of a few nuclei, and got values close to 0.1 eV. corresponding to a lifetime of  $10^{-14}$  seconds. This value agreed perfectly with the estimate given by Bohr in his paper on the compound nucleus, published in Nature. This made a great impression on everybody.

Weisskopf: Just a short remark about history. The concept of nuclear temperature and evaporation which was ascribed in some extent to me, actually should be

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ascribed also to Landau. Landau discussed the nuclear temperature first, and I learned about it from a paper by him.

*Bjørnholm*: How come that Bohr completely ignored the Pauli exclusion principle when proposing the compound-nucleus concept? Would you care to comment on how Bohr was reconciling the idea of a short mean free path with the idea that fermions should have a long mean free path inside the nucleus?

*Mottelson*: That is an interesting question. Apparently that kind of thinking was not understood—or was not used—in the period before the war, as far as I can gather. There are some notes by Niels Bohr after the discovery of shell structure in the late forties. I believe the notes are dated 1947 or 1948, in which he is trying to face up to the evidence for a long mean free path. He does not use that argument in those notes. He talks about a quantal non-localisation of the particles. But about six months later, in 1948 or 1949, he does refer to the argument that the Pauli principle will effectively prevent the correlations which would be involved in the short-range interaction.

Amaldi: I am sorry to speak again, but I should say something different from my dear friend Weisskopf. The first persons who spoke about nuclear temperature were André Debierne, A pupil and collaborator of Marie Sklodowska Curie and Henri Poincaré, in 1911–1912. Debierne published in part alone, in part in collaboration with Marie Sklodowska Curie, some interesting papers concerning the fact that the "atoms" of radioactive bodies "disintegrate at random". On various occasions, in particular at the end of a lecture that Debierne gave in January 1912 in front of the Société Francaise de Physique, he arrived at the conclusion that inside the atom there is an element of disorder, which causes the atom to pass through a great number of different states, in a very short instant of time, but that such an element of disorder is different from thermal agitation. As an example he suggested that the constituents of atoms are endowed with disordered movements similar to those of molecules of a gas inside a container. Commenting these views of Debierne, Poincaré noticed that this element of disorder should be described by statistical laws, and therefore by a "thermodynamics appropriate to the internal part of the atom", implying that one should define a temperature for the interior of the atom, which is not in thermal equilibrium with the external part. A short presentation of these ideas was also given by Marie Sklodowska Curie at the Solvay Conference of 1913. I would like to emphasize that all these ideas of Debierne and Poincaré were developed and presented in 1911–1912, i.e. before the existence of atomic nuclei was universally recognized. This is why these authors were speaking about atoms and not about nuclei.